Congruent numbers, quadratic forms and algebraic K-theory

We show that if a square-free and odd (respectively, even) positive integer n is a congruent number, then

$$\#\{(x,y,z)\in\mathbb{Z}^3|n=x^2+2y^2+32z^2\}=\#\{(x,y,z)\in\mathbb{Z}^3|n=2x^2+4y^2+9z^2-4yz\},$$

respectively,

$$\#\{(x,y,z)\in\mathbb{Z}^3|\frac{n}{2}=x^2+4y^2+32z^2\}=\#\{(x,y,z)\in\mathbb{Z}^3|\frac{n}{2}=4x^2+4y^2+9z^2-4yz\}.$$

If we assume that the weak Brich-Swinnerton-Dyer conjecture is true for the elliptic curves $E_n: y^2=x^3-n^2x$, then, conversely, these equalities imply that n is a congruent number.

We shall also discuss some applications of the proposed method. In particular, for a prime p, we show that if $p\equiv 1 \pmod 8$ is a congruent number, then the 8-rank of $K_2O_{\mathbb{Q}(\sqrt{p})}$ equals one, and if $p\equiv 1 \pmod{16}$ with $h(-p)\not\equiv h(-2p) \pmod{16}$, then $2p$ is not a congruent number.