Classification of non-commutative topological spaces which are not locally compact

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We will present a classification theorem for amenable simple stably projectionless C^*-algebras with generalized tracial rank one. With many decades' work, unital separable simple amenable $\mathcal{Z}$-stable C^*-algebras in the UCT class have been classified by the Elliott invariant. Non-unital case can be easily reduced to the unital case if the stablized C^*-algebras have a non-zero projection. However, there are many non-unital separable simple amenable C^*-algebras which are stably projectionless. In other words, $K_0(A)_+ = \{0\}$.

One of these simple C^*-algebras is what we called $\mathcal{Z}_0$. This C^*-algebras can be constructed as an inductive limit of so-called non-commutative finite CW complexes. It has exactly one tracial state and has the properties that $K_0(\mathcal{Z}_0) = \mathbb{Z}$, $K_0(A)_+ = \{0\}$ and $K_1(\mathcal{Z}_0) = \{0\}$. We will show that there is exactly one C*-algebra in the class of simple separable C^*-algebras with finite nuclear dimension and satisfies the UCT (up to isomorphism).

Let A and B be two separable simple C^*-algebras satisfying the UCT and have finite nuclear dimension. We show that $A \otimes \mathcal{Z}_0 \cong B \otimes \mathcal{Z}_0$ if and only if $\mathrm{Ell}(B \otimes \mathcal{Z}_0) = \mathrm{Ell}(B \otimes \mathcal{Z}_0)$. A class of simple separable C^*-algebras which are approximately sub-homogeneous whose spectra having bounded dimension is shown to exhaust all possible Elliott invariant for C^*-algebras of the form $A \otimes \mathcal{Z}_0$, where A is any finite separable simple amenable C^*-algebras. Suppose that A and B are two finite separable simple C^*-algebras with finite nuclear dimension satisfying the UCT such that both $K_0(A)$ and $K_0(B)$ are torsion (but arbitrary K_1). One consequence of the main results in this situation is that $A \cong B$ if and only if A and B have the isomorphic Elliott invariant.