Abstract

This talk is a review of the vertex-operator-algebraic constructions of and open problems on various tensor category structures on module categories for affine Lie algebras. I will review the pioneer construction by Kazhdan and Lusztig and a construction by Zhang(based on the logarithmic tensor category theory of Lepowsky, Zhang and myself) in the case that the sum of the level and the dual Coxeter number is not a nonnegative rational number. I will also review the construction by myself (as a special case for vertex operator algebras satisfying natural rationality conditions) and a construction by Finkelberg (after a gap was filled using the Verlinde formula in 2013) in the case that the level is a positive integer. I will discuss my recent construction, jointly with Creutzig and Yang, in the case that the level is admissible and will also present the conjectures on its rigidity, modularity and equivalence with a suitable quantum group tensor category.