Abstract

The search for superintegrable systems has demonstrated that quadratic and more generally polynomial algebras naturally occurs from conserved quantities of plethora of models. Those algebraic structures formed by higher order differential operators allow in the quantum setting to obtain formula for the degenerate energy spectrum for different Hamiltonians via representations. I will point out how the Racah algebras naturally arises in this context. I will discuss recent results, and a purely algebraic procedure based on the commutant of a subalgebra in the universal enveloping algebra of a given Lie algebra. I present the notion of algebraic Hamiltonians, constants of the motion and symmetry algebra. I will also discuss the current state of the classification of polynomials associated with commutant of Cartan subalgebra for semisimple algebras. I will point out the use of the Poisson-Lie setting and how it provides advantages.

The talk will focus on the case of the special linear Lie algebra sl(n), where an explicit basis for the commutant with respect to the Cartan subalgebra is obtained. I will highlight how with an appropriate realization, this provides an explicit connection with the generic superintegrable model on the

(n-1)-dimensional sphere and the related Racah algebra R(n). This will illustrate how this Racah algebra is in fact a particular case of wider algebraic structure taking the form of a polynomial algebra. I will provide explicit formula for the case of sl(3) and sl(4), their related polynomial algebras and their quantization.